DULWICH COLLEGE
FOUNDED 1619

Upper School entrance examination
SPECIMEN PAPER
Mathematics

Surname	
First Name	
Date of birth	
Current school	

Time allowed for this paper 90 minutes

Do all your written work on this paper, **showing all your working**.

Calculators are allowed

You should attempt all questions

Formulae

Quadratic Equation: $ax^2 + bx + c = 0, a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$ or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Volumes:

Cone:	$\frac{1}{3}\pi r^2 h$
Cylinder:	$\pi r^2 h$
Sphere:	$\frac{4}{3}\pi r^3$

Section A

The questions in this section test standard algebra (such as factorising, indices, solving equations, completing the square, algebraic manipulation), coordinate geometry & trigonometry

1. Simplify the following expressions as much as possible:

(i) $\dfrac{5x+25}{10x-20}$

Answer: _____

(ii) $\dfrac{2x^2+2x-12}{x^2-x-12}$

Answer: _____

(iii) $\dfrac{x^3-x^2}{x-1}$

Answer: _____

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2. Find the equation of the straight line through $(2,-4)$ and $(-9,7)$, writing your answer in the form $y=mx+c$.

Answer: _____

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3. Expand $(3+3\sqrt{2})(5-2\sqrt{2})$ and simplify your answer as far as possible.

Answer: _____

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4. Solve the following equations for x where $0^\circ \le x \le 180^\circ$ (**make sure you find all the angles in this range**):

(i) $\sin x = \sin 50^\circ$

Answer: $x =$ _____ $^\circ$

(ii) $\cos 2x = -\dfrac{\sqrt{3}}{2}$

Answer: $x =$ _____ $^\circ$

5.(a) Express each of the following as a power of 2 (i.e. in the form 2^x):

(i) $\dfrac{1}{16}$ Answer: _____

(ii) 64^a Answer: _____

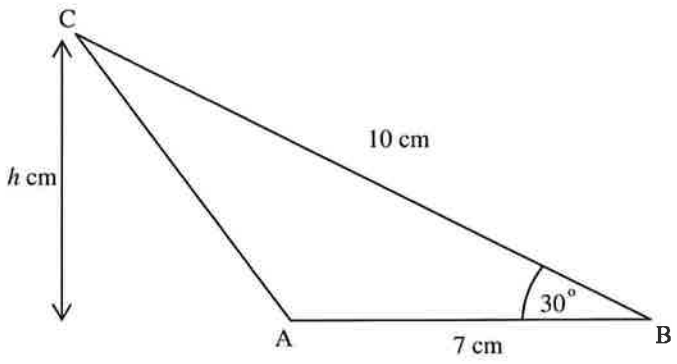
(iii) $\dfrac{8^b}{4^c}$ Answer: _____

(b) Solve the equation for x: $25^{3x} = \dfrac{1}{625}$.

Answer: _____

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6. In triangle ABC shown below (not drawn to scale), AB = 7 cm, BC = 10 cm and angle ABC = 30°. The perpendicular height of the triangle is h cm



Giving your answers to 3 significant figures where appropriate, calculate the:

(i) length AC;

Answer: AC = _____ cm

(ii) area of triangle ABC;

Answer: Area ABC = _____ cm^2

(iii) perpendicular height, h, of the triangle as shown in the diagram.

Answer: h = _____ cm

7. Solve the following equation for x, giving your answers to 1 decimal place:

$$\frac{2}{x-1} - \frac{3}{x+1} = 4$$

Answer: _____

Section B

The questions in this section are more stretching than those in section A and even though you may not have seen questions like these before they can all be answered with a little thought.

1.(i) Simplify the following $\frac{4}{x-3}+\frac{3x-3}{(x^2-x-6)}$ expressing your answer as a single fraction.

Answer: _____

(ii) Hence solve $\frac{4}{x-3}+\frac{3x-3}{(x^2-x-6)}=\frac{2-20x}{2x+4}$

Answer: _____

2. The lines with equations $y = 5x - 6$ and $10x + cy = 8$ are perpendicular. Find the value of c.

Answer: $c =$ _____

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3. Solve the simultaneous equations:

$$x - 2y = 1$$
$$x^2 - xy + y^2 = 1$$

Answer: _____

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4. You are given that $\tan x = \dfrac{\sin x}{\cos x}$.

Use this to solve: $\sin x + \cos x = 0$, for **all** x where $0° \leq x \leq 180°$.

Answer: $x =$ _____ °

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5. You are given that $x^3 - 2x^2 - 25x + 50 = (x-2)(ax^2 + bx + c)$ where a, b and c are integers.

(i) **Write down the** values of a, c.

Answer: $a =$ _____ $c =$ _____

(ii) Calculate the value of b.

Answer: $b =$ _____

(iii) Hence solve the equation $x^3 - 2x^2 - 25x + 50 = 0$.

Answer: $x =$ _____

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6. You are given that $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$.

Use this to expand the brackets and then simplify your answers in:

(i) $(1-y)^5$

Answer: $(1-y)^5 =$ _____

(ii) $(2x-y)^5$

Answer: $(2x-y)^5 =$ _____

(iii) $(x-\sqrt{2})^5(x+\sqrt{2})^5$

Answer: $(x-\sqrt{2})^5(x+\sqrt{2})^5 =$ _____

_____

Section C

This section contains questions on basic calculus (differentiation and integration). Only attempt this section if you have studied this material before.

1. Find $\frac{dy}{dx}$:

(i) $y = 5x^4 - x - 2$,

Answer: $\frac{dy}{dx} =$ _____

(ii) $y = \frac{1}{\sqrt{x}}$,

Answer: $\frac{dy}{dx} =$ _____

2.(i) Find $\int x^3 (x-4)\, dx$

Answer: _____

(ii) Evaluate $\int_0^1 \left(\frac{x^3 + x^2}{x^2} \right) dx$

Answer: _____

END OF EXAMINATION